

A Personal Philosophy of an Ex-Teacher on Mathematics Education: A Retrospective Integrated Account

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ABSTRACT

This is a retrospective integrated account of an ex-teacher's personal philosophy of mathematics education, which covers four main areas of concern: nature of mathematics and mathematics education; theories of how students learn mathematics; developments in mathematics education and implications for students with special needs; and specific types of difficulties in mathematics learning. In addition, the author has also included a section on the elements and priorities of mathematics curriculum for students with special needs. The author had taught mathematics for nine years in mainstream primary and secondary schools before he branched out into working with students with special needs and, much later, involved himself in training special education (SPED) teachers how to teach functional numeracy to students with moderate to serve disabilities.

Keywords: Curriculum, Disabilities, Learning, Mathematics, Special Needs

INTRODUCTION

I had taught mathematics for nine years in both primary and secondary schools from the mid-1980s to the mid-1990s, before I left the teaching service to join the then School Psychological Service (SPS), Ministry of Education, Singapore. After a two-year stint with SPS, I resigned to go into private practice as an educational therapist. My pre-service and in-service training at the then Institute of Education (now known as the National Institute of Education – an autonomous institution of Nanyang Technological University, Singapore) and my experience as a teacher had taught me many things or invaluable lessons, especially what it meant to me to become or being a professional educator (NOT just being a teacher) of mathematics.

This paper is the result of synthesizing a collection of my personal reflections that have been moulding my philosophy on mathematics education over time. I must emphasize at this juncture that it is very much my retrospective personal account of what I knew and understood about mathematics and mathematics education during those years when I was still a teacher. Hence, my cited sources in this paper would look out-dated, but let us not forget that I have written from a different chronosystemic period^{*}. Of course, I do not expect everyone to agree with me on everything that I have written or mentioned here. However, we could always choose to agree to be disagreeable so that the platform for academic debate can continue to help in advancing our knowledge about mathematics and mathematics education to the benefits of both teachers and students.

^{*} Researcher has chosen to write in hypothetical present tense to ease reading rather than moving back and forth between the past and the present that might result in confusion to the readers along the way.

NATURE OF MATHEMATICS AND MATHEMATICS EDUCATION

Three questions come into my mind when I have to discuss about the nature of mathematics and mathematics education. The first question asks “What is mathematics?” The second one being “How is mathematics to be defined?” The third and last question asks “Why should one be teaching/learning mathematics?” As an ex-teacher (and currently an academic training pre-service teacher trainees, in-service teachers as well as allied educators for learning and behavior support and school counselors), it is still important that I must have first a thorough understanding of what mathematics is before I can help my trainee teachers as well as students with special needs learn mathematics. My reflections (albeit retrospective) upon mathematics will determine to a great extent, my approach to the subject with my trainee teachers, allied educators and my students with special needs. In other words, my trainee teachers as well as students with special needs would benefit or be hindered according to the limits of my understanding of the scope of mathematics. For this reason, before I could consider how to help them learn mathematics, I should investigate the varied facets of the subject and begin to develop my own philosophy of what constitutes mathematics.

Returning to my first question asked: “What is mathematics?” While descriptions of mathematics abound, no single statement about the term seems quite sufficient (Baur & George, 1976). The term “mathematics” has different connotations for different people. The Australian Education Council (1990), for instance, defined mathematics as the science of space and number, but a more apt definition is that mathematics is the science of patterns. A mathematician – like a painter or a poet – is a master of patterns (Hardy, 1967). The mathematician seeks patterns in number, in space, in science, in computers, and in imagination, and mathematical theories help to explain the relations among patterns (Chinn & Ashcroft, 1992; Hardy, 1967). Applications of mathematics use these patterns to explain and predict natural phenomena. McEwan (1992) defined mathematics as the study of a group of sciences dealing with quantities, arrangements, magnitudes, and forms, and their relationships and attributes. According to that definition, there are four mathematical sciences: arithmetic, geometry, algebra, and calculus. More simply, mathematics might be defined as the science of quantity and space (Boyles & Contadino, 1998). In arithmetic, the symbol/quantity is fixed. For example, the symbol 8 represents the quantity of eight. In higher-order mathematics, such as algebra, the symbols are more arbitrary, and the spatial positions and orientation (e.g., once we change the orientation of 8 to ∞ , it represents infinity) become more important. From these selected definitions of mathematics, no matter how well each has stated, I, personally, find no description of mathematics that can be totally revealing (Baur & George, 1976).

“How then should mathematics be defined?” This is my second question asked. Baur and George (1976) provided a comprehensive definition of mathematics by examining it from various dimensions: historical, scientific, language, artistic, recreational, activity, and tool (see Baur & George, 1976, for more detail). The point here is which perspective(s) of mathematics, which mathematical dimension(s), should I relate to my trainees as well as my students with special needs? What relative importance should I give to each of these dimensions? When all these dimensions are put together, they provide me a more holistic understanding of the nature of mathematics. I can now see better as to which direction I should proceed with my trainee teachers as well as my students with special needs in mathematical pursuits. Responsibility for the mathematical development of my trainee teachers and my students with special needs provides a big challenge for me as a teacher/educator of mathematics.

The final question is “Why should one be teaching/learning mathematics?” In other words, why teach/learn mathematics? This is the initial question posed in the Cockcroft Report (Choat, 1983). Besides being an integral part of our general education, mathematics is useful for daily living, for instance, for commerce and for industry, because it provides a powerful, concise and unambiguous means of communication and because it also provides the means to explain and predict (Liebeck, 1984). In other words, the majority of people use mathematical ideas about number, space, movement, arrangement and chance in everyday life (Choat, 1983). Hence, mathematics can enhance our understanding of our world and the quality of our participation in society.

THEORIES OF HOW STUDENTS LEARN MATHEMATICS

When and how mathematical concepts and skills are acquired is somewhat theoretical to me. Piaget (1965) explained that they are developed over time as children pass from one phase of cognitive development to the next in more or less the same sequence: sensori-motor, pre-operational, concrete operational, and formal operations. However, the age at which a child reaches each phase varies, depending on each individual’s developmental rate. Besides Piagetian theory, there are also other theories of learning (e.g., Skemp, Bruner, and Dienes) that have also provided me an answer to the question how students learn mathematics.

Research studies (e.g., Boyles & Contadino, 1998; Bell, Costello, & Kuchermann, 1983), during my times as a teacher between 1980s and 1990s, had highlighted that the most effective learning situation is not the same for different individuals: the differences depends on intellectual factor (Maccoby & Jacklin, 1975; McKinnon, 1962; Pask, 1976a) and emotional factor (Head, 1980; Shadbolt & Leith, 1967), personality (Adorno, 1950; Krutetskii, 1976; Lewis & Ko, 1973), age (Leith & Bossett, 1967; Shadbolt & Leith, 1967), and possibly gender (Maccoby & Jacklin, 1975) and also, cultural background (Oxtoby & Smith, 1970).

In addition, motivation, dogmatism, social, political and religious values are also characteristics that, I feel, influence the way trainee teachers and students with special needs learn mathematics (Entwistle & Wilson, 1977). Moreover, teaching methods and styles (Bath & Knox, 1984) can affect mathematics learning. For example, it has been found that at primary level, exploratory methods are less effective with anxious students, but in secondary school, anxiety becomes less significant and it is the extravert students who gain most from exploratory situations (Trown, 1970; Trown & Leith, 1975). On the other hand, individual students’ learning strategies and styles can affect the way mathematics should be taught. For instance, some students are disposed to holistic strategies (“comprehension” learners) while others to serialism (“operation” learners). However, as a mathematics teacher before, I also found some students, who are described as versatile, able to act in either way according to the nature of the learning outcomes required (Pask, 1976b).

McGuinness (1985) and Boyles and Contadino (1998) have pointed out that mathematicians rarely teach mathematics, and seldom are they involved in the psychology of learning mathematics. According to Clements and Battista (1990), in reality, no one can “teach” mathematics. I believe effective mathematics teachers are those who can stimulate students to “learn” mathematics. Educational research (e.g., Cobb, 1988; Steffe & Cobb, 1988) that I had come across during my time as a teacher offers compelling evidence that students learn mathematics well only when they construct their own mathematical understanding through logical thinking.

In other words, what I see is that mathematics learning involves mathematical thinking. With mathematical thinking comes mathematical understanding. According to Burton (1984),

mathematical thinking is mathematical not because it is thinking about mathematics but because the operations on which it relies are mathematical operations. Its field of application is general. Mathematical thinking is used when tackling appropriate problems in any context area, although questions of a mathematical nature might more readily expose such thinking. A problem is appropriate to mathematical thinking when it provokes or responds to the use of the processes, specializing, conjecturing, generalizing, and convincing (see Burton, 1984, for more detail).

Being aware of the operations of mathematical thinking helps myself as a teacher/educator, my trainee teachers and my students with special needs to recognize our power in thinking about mathematical experiences. A sensitive appreciation of the dynamics of mathematical thinking draws attention to the decisive role of feelings in thinking and to the need to interlock action with thought and expression (Burton, 1984).

DEVELOPMENTS IN MATHEMATICS EDUCATION AND IMPLICATIONS FOR STUDENTS WITH SPECIAL NEEDS

The world was changing rapidly towards the end of the 20th century, especially after the end of World War II. Science and technology dramatically evolved and continue to evolve, and they are also changing mathematics. Hence, the mathematics that we learnt as students, that I taught as a teacher, and that the students of today learn, changes, too. It has been said that “some mathematics has become more important because technology requires it; some mathematics has become less important because technology replaces it; and some mathematics has become possible because technology allows it” (National Council of Teachers of Mathematics et al., n.d., p.C-5). Hence, the mathematics content must reflect current use of mathematics in our schools today (Trafton & Claus, 1994).

Most traditional mathematics instruction and curricula are based on the transmission, or absorption, view of teaching and learning. In this perspective, students passively absorb mathematical structures invented by others and recorded in texts or known by authoritative adults. Teaching consists of transmitting sets of established facts, skills and concepts to students (Clements & Battista, 1990).

Today, however, I see a paradigm shift from traditional instruction towards constructivist instruction in mathematics teaching/learning. Constructivism gives pre-eminent value to the development of my trainee teachers and my students’ personal mathematical ideas (Clements & Battista, 1990). It encourages them to use their own methods for solving problems. All methods devised by my trainee teachers or students, whom I once taught as a teacher, are valued, but my role as a teacher is to guide and support their invention of viable mathematical ideas through promoting the invention or adoption of more sophisticated techniques rather than transmit “correct” adult ways of doing mathematics. Moreover, as a constructivist teacher, I must be able to pose tasks that bring about appropriate conceptual reorganizations in my students whom I taught as a teacher. This constructivist approach depends on two important factors:

1. Knowledge of normal developmental sequence of students; learning of specific mathematical concepts and skills and the current individual students; and
2. Skills in structuring the intellectual and social climate of the learning environment so that students discuss, reflect on, and make sense of the tasks (Clements & Battista, 1990).

Until recent times, little modification of the mathematics education or curriculum was made for students with special needs. These students were expected to adapt to the mathematics

curriculum as it existed to take some sort of terminal sequence of courses developed for low-ability students. To reach the goals of developing or maximizing mathematical potential for all students with special needs requires the creation of a different curriculum for them and a least restrictive environment, in which teaching and learning are to take place, that are very different from much of current practice done in mainstream classroom (National Council of Teachers of Mathematics, 1991). In other words, this means that I need to rethink and redefine my ideas about the mathematics curriculum for students with special needs be they in mainstream or special class.

Implementing substantive educational change in mathematics curriculum to cater to students in SPED schools as well as those in mainstream schools is complex, for it involves rethinking long-established practices. In mathematics, I can see the difficulty of the changing process because of my view of mathematics resulted from my own experiences with it as a teacher. Change also involves confronting deeply held beliefs that make it difficult to perceive that things could be different. It challenges accepted views of what I do as I teach mathematics, and more so when dealing with students with special needs in a mainstream class. It involves accepting uncertainties associated with using less-structured teaching practices, departing from the traditional pedagogy. It also entails dealing with anxieties about current school policies, standard testing practices, and parental acceptance. The issues of change therefore are pronounced when working with students with special needs for they add complexity to teaching, decision-making, and program planning (Trafton & Claus, 1994).

Specific Types of Difficulties in Mathematics Learning

Mathematics to me (when I was a teacher during the period between mid-1980s and mid-1990s) is more than just picking up a pencil and computing an answer. It is the comprehension of mathematical concepts and the ability to apply them. Visual, language, memory, sequencing, and problem-solving skills, as well as the ability to comprehend spatial relationships, are all needed to solve a mathematical problem (Boyles & Contadino, 1998; Mercer, 1997).

Students who have trouble with mathematics may be experiencing difficulty in a range of challenges when working in these areas (see Mercer, 1997, for more detail) as briefly described below:

Visual Skills

Mathematics relies heavily on the use of visual skills, such as the recognition of numbers (Boyles & Contadino, 1998). Visual skills also affect the writing of problems. Many students with special needs have difficulty lining up columns of figures.

Language Skills

In many ways, mathematics has a language of its own. Before one can recognize what is being asked in story problems, one must learn certain key or signal words (Dunlop & Strobe, 1982). Teaching mathematical concepts requires that a lot of information be conveyed in a short period of time and students can easily become overwhelmed. Trying to understand unfamiliar terms imbedded in large amounts of information easily frustrate students with special needs.

Memory Skills

Because mathematics is a cumulative subject, each new concept builds on previously acquired information. This demand on the student's power of recall can become overwhelming (Miles & Miles, 1992). Students with poor automatic memory get stressed

when trying to learn and rapidly recall mathematical facts, especially multiplication tables. In higher-level mathematics, poor automatic memory interferes with the recall of formulas. Students may know when and where to apply a formula but unable to recall it. Multi-step mathematical functions require a particularly strong active working memory, and many students have difficulty in this area of memory (Mercer, 1997).

Sequencing Skills

Knowing how to do things in the correct order/sequence is an essential part of understanding mathematics: whether counting from 1 to 10 or solving complex problems in calculus. Knowing the order to follow is necessary to obtain the correct results. One step out of sequence in solving an algebraic equation results in an inaccurate answer. Remembering the sequence of steps required can pose a problem for students with temporal-sequential organization problems. Such students also have difficulty learning about time (Boyles & Contadino, 1998).

Problem-Solving Skills

Problem solving requires planning and organization before execution (Dunlop & Strobe, 1982). Using these so-called executive-function skills means that before an answer is given, a student must plan and organize a plan of attack. The ability to develop this plan is a critical part of being a good problem solver. Students with weak executive-function skills jump right in without fore thought and planning. They haphazardly and randomly attempt to solve problems. Solving a mathematical problem requires the same strategy. A plan is needed.

Spatial Relationships

The ability to visualize objects in space and remember their positions even when they have been moved about involves spatial reasoning (Miles & Miles, 1992). As mathematics becomes more abstract, this quality becomes more important. I see the importance of being able to translate words into mental images helps in understanding mathematical concepts. Beginning with the concrete before the abstract, helps many students make the spatial connections required in solving a problem.

Not Knowing One Does Not Know

It is not unusual for some students to sit in a mathematics class and think they understand what the teacher has said or taught. During the explanation, they are active participants. However, when they begin to do their assignment or take a test, they cannot work the problems. The problems may look different to them, and these students get confused and do not recognize what they need to do. This may lead to mathematics anxiety (Dodd, 1992; Hunt, 1985) and later phobia (Boyles & Contadino, 1998), if the problem becomes serious enough.

Elements and Priorities of Mathematics Curriculum for Students with Special Needs

Thornston et al. (1983) defined mathematics curriculum as the total set of general and specific mathematical content goals together with suggested means for achieving them. The emphasis is on nurturing the development of mathematical ideas in students.

However, having worked with students with special needs when I was still a mainstream school teacher, innovative or individualized methods became necessary. Generally, an individualized education plan (IEP) for each student with special needs becomes an essential aspect in modifying mathematics curriculum to bridge the gap between the regular curriculum and the special needs curriculum. This calls for, on my part, teacher originality and perseverance grounded in an understanding of the student's learning patterns and

knowledge of both standard and adaptive procedures for teaching mathematics that meet the goals of a balanced mathematics curriculum (Thornston et al., 1983).

Moreover, major efforts will be needed to create new materials and combine other existing materials in order to bring these areas together to satisfy the IEP. A continual assessment of students' progress and abilities is crucial to the IEP developed for the mathematics education of students with special needs. In any case, when dealing with such students in a mathematics class, as a teacher, I would have to be familiar with both the regular and special needs curricula.

Teaching mathematics to students with special needs is the most demanding aspect of teaching the subject. Hence, the development of an appropriate mathematics curriculum for students with special needs is essential. According to Thornston et al. (1983), such curriculum is based on many factors, the strongest of which, I personally believe, is probably tradition. However, the ideas and influence of professional and pressure groups have helped shape the general goals and the direction of the mathematics curriculum.

Though I am an ex-teacher, I have not forgotten that it has always been difficult to state the general goals of the mathematics curriculum for students with special needs. The statement of general goals for the mathematics curriculum is the central issue in the back-to-the-basics movement (Thornston et al., 1983). This issue is not new, but it is exceedingly difficult to deal with. Part of the problem as I see it lies in the way general goals are stated and part lies in deciding the relative emphasis for each goal.

Most of the teachers during my times would agree with me that students should be able to compute with whole numbers, fractions and decimals; to apply basic geometric ideas; to carry out everyday measurement situations; and to solve problems. Others add the additional objective of developing positive attitudes towards mathematics learning.

To meet these expectations and demands effectively as a mathematics teacher then, I saw the need to be skillful and knowledgeable. There are several important elements that I have to consider in order to deal effectively with students with special needs – be they in mainstream or SPED school:

Knowing Students

Every teacher needs to be sensitive to the individual students' interests. All of this is most helpful to the teacher in planning mathematics instruction. Such information can allow the teacher to use familiar analogies in teaching concepts and skills, a type of instruction effective with students of all diverse abilities. Moreover, it also helps the teacher understand students' learning capabilities, and strengthens the instruction to be offered in either a one-to-one or group situation. Good student orientation is important to all teachers in keeping records on a student's growth and learning patterns.

Knowing How to Manage a Classroom

This includes instructional sequences, monitoring on-task behaviors, and dealing with discipline situations. In addition, it includes a teacher's ability to adapt classroom flow to meet students with special needs without losing others in the class, focus students' energies in fruitful directions and provide boundaries for students' behaviors. It is important to note that students with special needs often do not perform well in a hectic, unfocused classroom situation.

Knowing the Relevant Mathematics

It is equally important for teachers to know the necessary mathematics content in order to successfully implement curricular objectives and manage classroom learning. The National Council of Supervisors of Mathematics (1977) has listed ten basic skills essential for inclusion in the mathematics content for learning:

- a. Alertness to the reasonableness of results;
- b. Application of mathematics to everyday situations;
- c. Appropriate computational skills;
- d. Computer literacy;
- e. Estimation and approximation;
- f. Geometry;
- g. Measurement;
- h. Problem solving;
- i. Reading, interpreting and constructing tables, charts and graphs; and
- j. Using mathematics to predict.

Understanding the Student's Special Needs

Teachers have to adapt the mathematics program to each student's needs. They need to know as much as possible about the various learning and behavioral challenges students with special needs have. In addition, teachers need to know enough about co-morbid disabilities, e.g., dyslexia and attention deficit/hyperactivity disorder (ADHD), to identify problems that may arise during their contact with such students. When problems do occur, they are prepared to refer a student for special diagnosis or help. Such an understanding of students' special needs prepares teachers to perform a more effective job of planning instructional sequences in mathematics.

Understanding the Student with Special Needs

It is important for every teacher to know the limits and special characteristics of each student's disabilities. The student's educational history will provide information concerning effective instructional approaches, gaps in content knowledge, and levels of achievement and aptitude. The family history will also provide valuable information about the student's self-image, the degree to which the student may have been protected or challenged, and the attitude of the family towards mathematics. Finally, teachers need to observe the quality and quantity of the interactions between the student and his/her peers. This reveals the student's approach to life, attitudes about schoolwork, and feelings about mathematics.

Understanding Mathematical Learning Patterns

Teachers need to understand the patterns of mathematics instruction that lead to effective learning (Bath & Knox, 1984). Knowledge of developmental patterns in counting, number recognition, and measurement skills can help teachers to plan and sequence lessons that promote mental models as well as concrete understanding of the concepts.

Knowing and Understanding Concrete Teaching Aids for Mathematics

An effective teacher is aware of the different types of teaching aids available and also able to effectively use them. Effective use must take into account the students' level of mathematical maturity and development.

CONCLUSION

In my concluding remarks, there are two further questions to be asked here. The first question concerns about the purpose of teaching/learning mathematics. To put it in another way, “What is the purpose for knowing mathematics?” Trafton and Claus (1994) have explained the purpose: it is “to use mathematics to solve problems, to represent ideas in various ways, to discern patterns, to make sense of new situations, to make and justify conjectures, and to communicate one’s ideas to others” (p.22).

The second question concerns how I, being an ex-teacher, who had taught mathematics in mainstream schools, can effectively impart the knowledge of mathematics to my trainee teachers as well as allied educators for learning and behavior support so that they, in turn, can help students with special needs should they encounter them. To be an effective mathematics teacher of students with special needs, it is important for me to state clearly that teaching mathematics to such students requires a very careful analysis of the language I employ in my own teaching, of the utilization of sensory channels (i.e., visual-spatial, vocal-auditory-sequential, and kinesthetic-tactile) and yet of the differentiation in relation to the students’ individual learning needs (e.g., differences in the ways students with special needs approach mathematical problems).

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