STUDENTS' ERRORS IN SOLVING UNDEFINED PROBLEM IN ANALYTIC GEOMETRY IN SPACE: A CASE STUDY BASED ON ANALOGICAL REASONING

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ABSTRACT

In this study, we used a type of "undefined problem" in analytic geometry in space and analogical reasoning as tools to find out students' errors in learning mathematics; here, we defined an undefined problem in geometry as a problem that has the infinite number of objects satisfying the insufficient conditions of the problem. Participants were 309 grade 12th students in schools of Vietnam, in which the students were assigned to one undefined problem to solve. The results showed that students revealed many errors in solving undefined problems in geometry and the use of analogical reasoning was one of causes of their errors.

Keywords: Error in solving problem, analogical reasoning, undefined problem in geometry, mathematics education.

INTRODUCTION

Topic "Errors of students in learning science and mathematics" has been the one which many educators from different countries were interested in. Marzano introduced how to analyze errors; Loc & Hoc (2014), Hoc (2014) presented the results of the investigation on students' errors in learning calculus; Loc & Kha (2015), Kha (2015) showed errors which students often committed in analytic geometry.Also studying errors of students, in this study, we wanted to use "an undefined problem in Geometry" and analogical reasoning to recognize and explain students' errors.

CONCEPTS

Error in Solving Problem

Definition 1: Error in solving problem is an error caused by improperly implementing mathematical rules; by applying the incorrect mathematical formulas, mathematical theorems; or by misunderstanding concepts, theorems; by misunderstanding an assignment, or by making mistake in calculation and presenting problem solution (Loc & Hoc, 2014; Loc & Kha, 2015).

Analogical Reasoning

Definition 2: According to Visual Mathematics Dictionary, analogical reasoning is"a type of reasoning in which it is assumed that if there is a similarity or sameness between two problems or methods in some aspects, they may be alike in other aspects. This is not a reliable reasoning because it is not always true".

Undefined Problem in Geometry

Definition 3: In this study, we define an undefined problem in geometry as a problem that has the infinite number of objects satisfying the insufficient conditions of the problem. For

example, in the case of the problem "In plane Oxy, write the equation of a circle with the center I (1; 2)"; it is an undefined problem because there was the infinite number of circles that their center is I.

STATEMENT OF RESEARCH PROBLEM

In Vietnam, in the process of learning analytic geometry in space, Grade 12th students are familiar with the type of task: Write the equation of a straight line with given conditions which are enough to define the straight line. For this type of task, students find it not difficult to solve because it is routine problem. However, in the case of an undefined problem such as: "In Oxyz, write the equation of a straight line with given facts insufficient to define the straight line", how will students to find its solution? Which errors happen to the students? About the cause of the students' errors, we have the following hypothesis:

H: *Because the students use the strategies for solving analogical problem in the plane Oxy to solve the problem in space Oxyz, thus they will make errors.*

PURPOSE OF RESEARCH

The purpose of this study was to verify the hypothesis H as above. The type of undefined problem used to verify Hwas:

> "In space Oxyz, write the parametric equation of a straight line Δ passing through $M(x_0, y_0, z_0)$ and perpendicular to the straight line passing through two distinct points $A_1(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$". (P1)

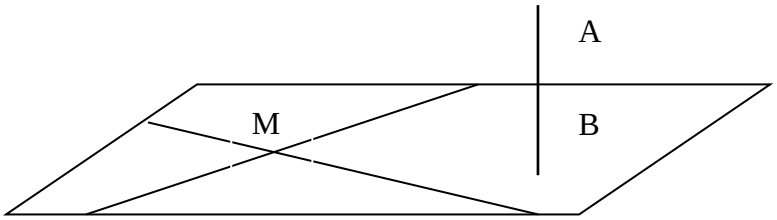

Figure 1. The infinite number of straight lines through M and perpendicular to (AB)

The problem (P1) is the kind of undefined one because there is the infinite number of the straight lines passing through $M(x_0, y_0, z_0)$ and perpendicular to the straight line AB; therefore, the parametric equation of Δ is not defined. In analytic geometry in plane, there is the analogical type of problem as follows:

> "In plane Oxy, write the parametric equation of a straight line Δ passing through M (x_0, y_0) and perpendicular to the straight line passing through two points $A(x_1, y_1)$ and $B(x_2,y_2)$". (P2)
>
> For (P2), we can use the following strategies to solve:

The strategy S_1: We have: $\overrightarrow{AB} = (a,b)$, where a = x_2-x_1; b = y_2-y_1. It implies that $\vec{u} = (b,-a)$ is a direction vector of Δ.

Therefore, the parametric equation of a straight line Δ in form: $\begin{cases} x = x_0 + bt \\ y = y_0 + (-a)t \end{cases}$

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The strategy S_2: Find out the coordinates of N on the straight line AB such that MN is perpendicular to AB. So that $\overrightarrow{MN}$ is a direction vector of Δ .

We predict that students will committed errors because they usestrategies analogical to S_1 or S_2 to solve (P1).

METHODOLOGY

The particular undefined problem assigned to students:

"Write the parametric straight line Δ passing through M(1;3;-2) and perpendicular to the straight line containing two points A(-1;-2;1) and B(3;1; -2)". (P3)

Subjects: 309 grade 12^{th} students (academic year 2014 -2015) from two schools: The High school "Thực hành Sư phạm" (Can Tho University, Vietnam) and the High school "Nguyễn Thị Định" (Bến Tre province, Vietnam). (see Table 1).

Data collecting and analyzing: These participants were assigned the problem (P3) to solve. After the students finishing solving the above problem, we analyzed their solutions to the problem, and interviewed 4 students to know their opinions on how to solve the problem.

Table 1. Students investigated from schools

School	Class	The number of students (N=309)
High school Nguyễn Thị Định (Bến Tre province)	12T1, 12A2, 12A4, 12A7, 12A8	165
High school Thực hành Sư phạm (Cantho University)	12A1, 12A2, 12B1, 12B2	144

RESULTS AND DISCUSSION

Analyzing the solutions of students

Based on the students' the solutions to the given problem (P3), we could summarized the strategies which students used as follows (see Table 2):

Strategy S_I: (analogical to S_1) : From $\overrightarrow{AB}$ = (4, 3. -3), students commuted coordinates of $\overrightarrow{AB}$ to have a direction vector of Δ .

Strategy S_II: (analogical to S_2)*:* They found N on the straight line AB such that MN is perpendicular to AB, then $\overrightarrow{MN}$ is the direction vector of Δ .

Strategy S_III: They argued that through a given point M, there is the infinite number of straight lines perpendicular to the straight line AB.

Strategy S _ IV: Many students used the wrong strategy as follows: They considered $\overrightarrow{AB}$ as a direction vector of Δ .

Strategy _V: Some students wrote that the equation of the straight line Δ was the equation of the plane through M and perpendicular to the straight line AB.

Table 2. Students' strategies to solve the problem

Strategy	The number of students (N=309)	%
S_I	74	23.9
S_II	29	9.39
S_III	1	0.32
S_IV	175	56.65
S_V	5	1.62
No answer	25	8.08

Table 2 showed that over 56% of students had the same opinion that a direction vector of AB is also the one of the straight line Δ ; it is a wrong idea by their knowledge gap. About 34% of students used the strategies S_I and S_II to solve the problem, so they committed errors; these errors were caused by using analogical reasoning. This fact allowed us to reach the conclusion that several errors of students in solving mathematical problems were caused by their use of analogical reasoning; and students found it difficult to solve undefined problems in analytic geometry.

The interviewing results

Through interviews, we found that the students based on the analogical problem in plane Oxy to find out the solution to the problem in space Oxyz. So, they made errors in solving the target problem. For instance, when they believed that in space, like in plane, if two straight lines are perpendicular to each other, then a direction vector of the one is also a normal vector of another one ; it is an error caused by analogy.

CONCLUSION

The results showed that undefined problems were not easy for students to solve. Students' errors could be revealed when they approached this type of problem. One thing is necessary to notice that using analogical reasoning was one of error causes; therefore, in process of teaching a mathematical content, the teacher can make use of undefined problems as a tool for evaluating learning actual understanding of his students, and know how to prevent students' errors due to analogical reasoning.

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